

Section 8.4: Matrix Algebra

Scalar Multiplication

Multiply each entry in the matrix by the given scalar.

Example:

$$A = \begin{bmatrix} 3 & 7 \\ 9 & 10 \end{bmatrix}$$

$$\begin{aligned} 2A &= \begin{bmatrix} 2 \cdot 3 & 2 \cdot 7 \\ 2 \cdot 9 & 2 \cdot 10 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 14 \\ 18 & 20 \end{bmatrix} \end{aligned}$$

How to add two matrices

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

A diagram showing the addition of two 2x2 matrices. Yellow circles highlight the numbers 3, 4, 8, and 0 from the first matrix, and 4, 1, 0, and -9 from the second matrix. A yellow arrow points from the sum 3+4=7 to the resulting matrix entry 7.

These are the calculations:

$$\begin{array}{|c|c|} \hline 3+4=7 & 8+0=8 \\ \hline \end{array}$$

$$\begin{array}{|c|c|} \hline 4+1=5 & 6-9=-3 \\ \hline \end{array}$$

Matrix Subtraction

$$\begin{bmatrix} -1 & 2 & 0 \\ 4 & 1 & 10 \end{bmatrix} - \begin{bmatrix} 3 & 2 & 4 \\ 0 & 8 & 16 \end{bmatrix} = \begin{bmatrix} -1-3 & 2-2 & 0-4 \\ 4-0 & 1-8 & 10-16 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 0 & -4 \\ 4 & -7 & -6 \end{bmatrix}$$

final answer

$$\begin{bmatrix} -4 & 3 \\ 1 & 9 \end{bmatrix} + \begin{bmatrix} 5 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 4 & 1 \end{bmatrix}$$

2 x 2 matrix **3 x 3 matrix**

The addition is undefined.

You can write as your answer:

undefined

Matrix Multiplication

$$(m \times n) \cdot (n \times k) = (m \times k)$$

product is defined

Here is an example of the product of a 2×3 matrix and a 3×2 matrix, the result is a 2×2 matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 \end{bmatrix}$$

$(1 \times 7 + 2 \times 9 + 3 \times 11)$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \end{bmatrix}$$

$(1 \times 8 + 2 \times 10 + 3 \times 12)$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 \end{bmatrix}$$

$(4 \times 7 + 5 \times 9 + 6 \times 11)$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix}$$

$(5 \times 8 + 5 \times 10 + 6 \times 12)$

Here is an example of the product of a 2×2 matrix with a 2×2 matrix, the result is a 2×2 matrix

Matrix Multiplication

$$\begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 5 \\ 3 & 7 \end{bmatrix} = \begin{bmatrix} 3+12 & 15+28 \\ 2+3 & 10+7 \end{bmatrix}$$

Matrix 1 Matrix 2
= $\begin{bmatrix} 15 & 43 \\ 5 & 17 \end{bmatrix}$

Resultant
Matrix

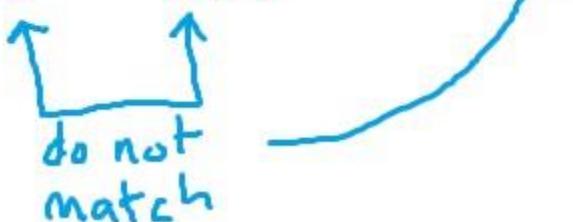
If the "inner" numbers don't match, then the product is undefined.

$$\begin{bmatrix} 5 & 6 & 1 \\ 0 & 5 & 7 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -3 & -1 & 0 \\ -2 & 8 & 1 \end{bmatrix}$$

undefined

answer

3×3 2×3



Use the following matrices to answer all the problems in this section.

$A = \begin{bmatrix} -2 & 4 \\ 3 & 1 \end{bmatrix}$	$B = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$	$C = \begin{bmatrix} 1 & 0 & -1 \\ 7 & 2 & 4 \\ 2 & 3 & -5 \end{bmatrix}$
$D = \begin{bmatrix} 3 & 2 & 0 \\ 4 & -1 & 3 \\ 1 & -5 & 6 \end{bmatrix}$	$E = \begin{bmatrix} -3 & 2 & 2 \\ 5 & -1 & 3 \end{bmatrix}$	$F = \begin{bmatrix} 1 & -2 \\ 5 & 2 \\ 6 & 7 \end{bmatrix}$

Perform the indicated operation

- | | | | |
|------------|-------------|-----------|-----------|
| 1) 3A | 2) 5B | 3) -2F | 4) -4E |
| 5) A + B | 6) C + D | 7) 2A+3B | 8) 2C +6D |
| 9) 3A – 2B | 10) 5C – 4D | 11) A + C | 12) B + D |
| 13) D + E | 14) E + F | 15) AB | 16) BA |
| 17) FA | 18) EF | 19) CD | 20) DC |
| 21) BC | 22) AD | 23) FE | 24) AF |